Some Hidden Stochastic Games that Have a Value





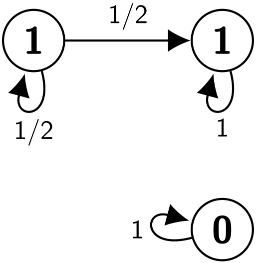
A. Asadi[1] K. Chatterjee[1] D. Lurie[2] **R. Saona**[3] B. Ziliotto[4]

[1]Institute of Science and Technology Austria (ISTA)
[2]Paris-Dauphine University (PSL)
[3]London School of Economics and Political Science (LSE)
[4]CNRS at Toulouse School of Economics (TSE)

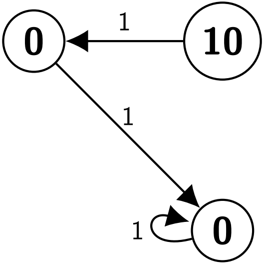
30 years of Game Theory at IHP — Oct 2025

Can I discretize
my continuous space and study
limit objectives in the game?

Simple blind MDP

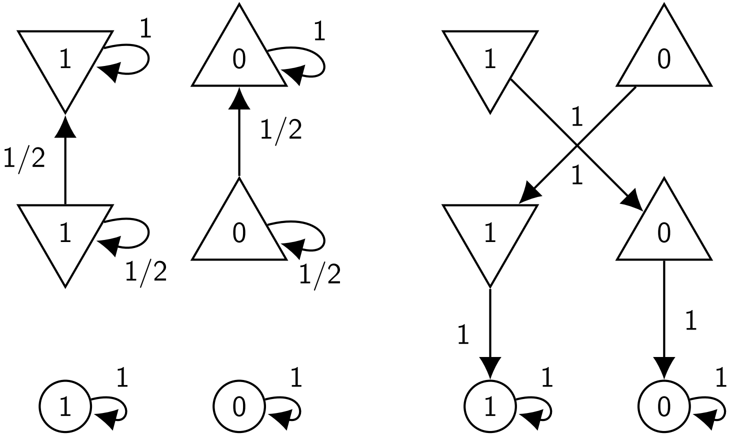


(a) Safe



(b) Risk

Blind MDP with actions (a) `Safe` and (b) `Risk`.



(a) Wait (b) Commit

Single-controller stochastic game with actions (a) `Wait` and (b) `Commit`.

Difficulty: Absorbing states

Difficulty:
Absorbing states can **accumulate arbitrarily small contributions**. So, the player(s) behaviour depends on nonapproximable effects because, in the limit value, they are infinitely patient.

Definitions

Blind Stochastic Games

A Blind Stochastic Game is a tuple $\Gamma = (\mathcal{K}, \mathcal{I}, \mathcal{J}, \delta, r, s_1)$ where

- $\mathcal{K}$ is a finite set of **states**.
- $\mathcal{I}$ and $\mathcal{J}$ are finite sets of **actions** for each player.
- $\delta \colon \mathcal{K} \times \mathcal{I} \times \mathcal{J} \to \Delta(\mathcal{K})$ is a probabilistic **transition** function.
- $r \colon \mathcal{K} \to \mathbb{R}$ is a **reward** function.
- $k_1 \in \mathcal{K}$ is an **initial state**.

Model. Players know the game Γ. They play simultaneously and observe each others actions.
Therefore, **they have the same belief** over the current state.

Limit Value

Denote σ and τ general strategies for the players.
For $\lambda \in (0,1)$, the λ-objective of the players is to optimize

$$\gamma_\lambda(\sigma,\tau) := \mathbb{E}_{k_1}^{\sigma,\tau}\left(\lambda \sum_{t=1}^{\infty} (1-\lambda)^{t-1}\, r(K_t)\right).$$

The discounted value is defined as

$$\mathrm{val}_\lambda := \min_\sigma \max_\tau \gamma_\lambda(\sigma,\tau) = \max_\tau \min_\sigma \gamma_\lambda(\sigma,\tau).$$

The (limit) value is defined as

$$\mathrm{val} := \lim_{\lambda \to 0^+} \mathrm{val}_\lambda .$$

Previous results

Mertens' Conjecture

Conjecture (1987, International Congress of Mathematics)

In every (zero-sum) stochastic game, the (limit) value exists.

Proven in many special cases of stochastic games.

Limit Value: Existence

Theorem (2002, Rosenberg & Solan & Vieille, Annals of Statistics)

Every blind 1-player stochastic game has a (limit) value.

Limit Value: Nonexistence

Theorem (2016, Bruno Ziliotto, Annals of Probability)

There exists a blind stochastic game where the (limit) value does not exist.

Theorem (2003, Madani & Hanks & Condon, Artificial Intelligence)

The problem of recognizing bounds ε-apart from the (limit) value of blind MDPs is undecidable.

Ergodic transitions

Ergodicity: Forgetting where you come from

In Markov Chains, an ergodic transition probability P satisfies

$$\lim_{n\to\infty} P^n = \mathbb{1}\mu^\top .$$

Equivalently, for all $p \in \Delta(\mathcal{K})$, we have that

$$p^\top \lim_{n\to\infty} P^n = \mu^\top .$$

In particular, $k, \widetilde{k}, k' \in \mathcal{K}$

$$\lim_{n\to\infty} \left| P^n_{k,k'} - P^n_{\widetilde{k},k'} \right| = 0 .$$

Coefficient of Ergodicity

Definition (Coefficient of Ergodicity)

Given a matrix $P \in \mathbb{R}^{\mathcal{K} \times \mathcal{K}}$, define

$$\operatorname{erg}(P) := \max_{k, \widetilde{k} \in \mathcal{K}} \sum_{k' \in \mathcal{K}} \left| P_{k,k'} - P_{\widetilde{k},k'} \right| .$$

Note that

- $\operatorname{erg}(PQ) \leq \operatorname{erg}(P) \operatorname{erg}(Q)$.
- $\operatorname{erg}(P) = 0$ if and only if $P = \mathbb{1}\mu^\top$.

Ergodic Blind Stochastic Games

Definition (Ergodic blind stochastic game)

For all action pairs $(i, j) \in \mathcal{I} \times \mathcal{J}$,

$$\operatorname{erg}\left(P(i,j)\right) < 1\,.$$

Lemma

Consider an ergodic blind stochastic game. For all $\varepsilon > 0$*, there exists an integer* n_ε *such that,*
for all $n \geq n_\varepsilon$ *and tuples of action pairs* $(i_1, j_1, \ldots, i_n, j_n)$*,*

$$\operatorname{erg}\left(P(i_1, j_1) \cdots P(i_n, j_n)\right) \leq \varepsilon\,.$$

Intuitively, the current belief is approximated by considering only the last n_ε actions:
no need to remember your initial distribution!

Our Contributions

Limit Value: Existence

Theorem

Every ergodic blind stochastic game has a limit value.

Proof sketch.

- Construct a finite stochastic game based on n_ε steps at a time.
- Belief dynamics remain close between
  the original and approximated model.
- Finite-stage payoff remain close between the models.

□

Limit Value: Approximability

Theorem

Approxmating the limit value of an ergodic blind stochastic game can be done in 2-EXPSPACE.

Proof sketch.

- The previous construction requires 2-EXP states.
- Approximating the limit value can be done by solving a sentence of the first order theory of the reals, which is PSPACE on the input.

□

Limit Value: Undecidability

Theorem

The problem of recognizing lower and upper bounds of the limit value of ergodic blind MDPs is undecidable.

Proof sketch.

- Consider an arbitrary blind MDP.
- Add a positive transition to a new state and a restart action.
- These modifications do not change the limit value, because the controller is infinitely patient.
- Remarkably, the transitions are now ergodic!

□

Summary of Contributions

Blind Class	Existence	Approximation	Exact
SGs	No	–	–
Ergodic SGs	**Yes**	**2-EXPSPACE**	**Undecidable**
MDPs	Yes	Undecidable	Undecidable
Ergodic MDPs	Yes	**2-EXPSAPCE**	**Undecidable**

Summary of results

Thank you!